ENGINEERING MATHEMATICS

***Time**: Three Hours*

***Maximum Marks**: 100*

Answer five questions, taking ANY TWO from Group A, any two from Group B and all from Group C.

All parts of a question (a, b, etc.) should be answered at one place.

Answer should be brief and to-the-point and be supplemented with neat sketches. Unnecessary long answer may result in loss of marks.

Any missing or wrong data may be assumed suitably giving proper justification.

Figures on the right-hand side margin indicate full marks.

Group A

1. (a) If $y = \tan^{-1} x$ y=, show that 8

$$(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$$

(b) Test the convergence of the series 6

$$\frac{1^2}{2} + \frac{2^2}{2^2} + \frac{3^2}{2^3} + ... + \frac{n^2}{2^n} + ...$$

(c) If $I_{m,n} = \int \cos^m x \sin nx dx$, prove that 6

$$I_{m.n} = -\frac{\cos^m x \cos nx}{m+n} + \frac{m}{m+n} I_{m-1,n-1}$$

Hence or otherwise, evaluate

$$\int_0^{\pi/2} \cos^5 x \sin 3x dx$$

2. (a) Find the volume generated by revolving about the axis of x. The area bounded by the curves 8

$$x^2 + y^2 = 25, 3x - 4y = 0, y = 0$$

lying in the first quadrant.

(b) If $u = xf(y/x) + g(y/x)$ then prove that 6

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0$$

(c) Divide 24 into three parts such that the continued product of the first, the square of the second and the cube of the third may be maximum. 6

3. (a) Determine whether the vectors 8

$$v_1 = (1,-2,3); v_2 = (5,6,-1); v_3 = (3,2,1)$$ form a linearly dependent or linearly independent set.

(b) Determine the value of 'a' such that divergence of 6

$$f(x+3y)i + (y-2z)j + (x+az)k$$ vanishes.

(c) Show that the vector field $\vec{F} = 2x(y^2+z^3)i + 2x^2yj + 3x^2z^2k$ is conservative. Find its scalar potential and the work done by it in moving a particle from (-1,2,1) to (2,3,4). 6

4. (a) Verify the Green's theorem for $f(x,y) = e^{-x}\sin y, g(x,y) = e^{-x}\cos y$ and the contour C is the square with vertices at (0,0), (π/2, 0), (π/2, π/2) (0, π/2). 6

(b) Solve the following system of equations by using Gauss elimination method 8

$$x - y + 2z = -8$$
$$x + y + z = -2$$
$$2x - 2y + 3z = -20$$

(c) For what values of λ and μ do the system of equations 6

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

have (i) no solution (ii) unique solution (iii) more than one solution?

Group B

5. (a) Is the differential equation $\left(1+e^{x/y}\right)dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$ exact? Find the general solution. 8

(b) Show that differential equation 6

$$(\cos y - \cos x)dx + (e^y - x\sin y)dy = 0$$

is exact, and find the particular solution such that y(π) = 0, that is y = 0 at x = π.

(c) Solve the differential equation 6

$$\frac{dy}{dx} = (4x + y + 1)^2, if\ \ y(0) = 1$$

6\. (a) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$ 8

(b) Solve the first order partial diff. q. 6

$$x(y^2 - z^2)\frac{\partial z}{\partial x} + y(z^2 - x^2)\frac{\partial z}{\partial y} = z(x^2 - y^2)$$

(c) Form a partial differential equation by eliminating the arbitrary function ϕ from 6

$$\phi(x^2 + y^2 + z^2, z^2 - 2xy) = 0$$

7\. (a) Use difference operator technique to get missing values in the following table: 6

x	40	50	55	60	55
y	3.0	-	2.0	-	-2.4

(b) Given f(2) = 4 and f(2.5) = 5.5. Find the linear interpolating polynomial using (i) Lagrange's interpolation (ii) Newton's divided difference interpolation. Hence, find an approximation value of f(2.2). 8

(c) Find the first and second derivative of the functions tabulated below at x = 0.40: 6

x	0.40	0.50	0.60	0.70	0.80
y	1.5836	1.7974	2.0442	2.3275	2.6511

8\. (a) Estimate the value of π, correct to the three decimal places, from $\int_0^1 \frac{1}{1+x^2}dx$ by using Simpson's one third rule when number of sub intervals is 4. 8

(b) Find the mean and standard deviation of the following data: 6

19.7, 21.5, 22.5, 22.2, 22.6

21.9, 20.5, 19.3, 19.9, 21.7

22.8, 23.2, 21.4, 20.8, 19.4

22.0, 23.0, 21.1, 20.9, 21.3

(c) A company generally purchases large lots of a certain kind of electronic 6 device. A method is used that rejects a lot if two or more defective units are found in a random sample of 100 units.

(i) what is the probability of rejecting a lot that 1% is defective?

(ii) what is the probability of accepting a lot that is 5% defective?

Group C

9. Answer the following in brief: 20

(i) If Δ and ∇ are the forward and backward difference operators respectively, then the value of $\Delta - \nabla$ is ________.

(ii) State Roll's theorem.

(iii) Discuss the applicability of Lagrange's mean value theorem for $f(x) = (1/x)\ln \quad [-1,1]$

(iv) Find the directional derivative of $f(x, y, z) = xy^2 + 4xyz + z^3$ at (1,2,3) in the direction of $3i + 4j - 5k$.

(v) Show that $E \equiv 1 + \Delta$ and $\Delta = \nabla(1-\nabla)^{-1}$

(vi) Find $div\left(\frac{\bar{r}}{|\bar{r}|}\right), where\, r = xi + yj + zk$

(vii) Change the order of equation in $\int_0^a \int_{mx}^{ix} f(xy)dydx$

(viii) Under what conditions the differential equation $M(x, y)dx + N(x, y)dy = 0$ will be exact and why?

(ix) Find the length of the arc of semi-cubical parabola $ay^2 = x^3$ from the vertex to the point (a,a).

(x) Find the rank of matrix

$$\begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$$

using the concept of linear independency.

(Refer our course material for answers)